

The probability of iterated conditionals

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This research was supported in part by a grant from the EURO-XPRAG Network. We are grateful to Andy Fugard and an anonymous referee for valuable comments on a previous version of this paper.

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Keywords: Indicative conditionals; iterated conditionals; decision-theoretic account of reasoning; probability; defective truth-table

Abstract

Iterated conditionals of the form *If p, then if q, r* are an important topic in philosophical logic. In recent years, psychologists have gained much knowledge about how people understand simple conditionals, but there are virtually no published psychological studies of iterated conditionals. This paper presents experimental evidence from a study comparing the iterated form, *If p, then if q, then r* with the ‘imported’, non-iterated form, *If p and q, then r*, using a probability evaluation task and a truth table task, and taking into account qualitative individual differences. This allows us to critically contrast philosophical and psychological approaches that make diverging predictions regarding the interpretation of these forms. The results strongly support the probabilistic Adams conditional, and the ‘new paradigm’ that takes this conditional as a starting point.

The Probability of Iterated Conditionals

1. Introduction: Iterated conditionals

Conditional (*if then*) sentences are of vast importance in cognitive science. Much of our knowledge can be expressed by sentences of this form, and they play a key role in modeling many kinds of thinking, such as planning and decision-making. For this reason, conditionals have been a central area of investigation in philosophy, linguistics and psychology. Yet in spite of their centrality, hardly anything about conditionals is uncontroversial. There is not even agreement about basic things like the conditions under which they are true, or which principles for conditional reasoning are valid.

In the past one or two decades, however, psychologists have come to agree that almost all useful inference, in science and everyday affairs, depends on propositions that are uncertain. According to some, this does not square well with theories of reasoning that are founded on classical extensional logic. Instead, they take a probabilistic approach, which is one of the main features of the ‘new paradigm’ (Elqayam & Over, 2013; Evans, 2012; Oaksford & Chater, 2009; Over, 2009). Others, especially in mental model theory, are happy to stick to the binary tradition, where propositions are just classified as true or false (see Johnson-Laird & Byrne, 2002, and their ‘fundamental principle of truth’). Conditionals take a central place in this ongoing debate. A prominent experimental finding is that people judge the probability of the indicative conditional, $P(\text{If } p, q)$, to be the conditional probability of q given p , $P(q|p)$, commonly referred to as ‘the Equation’ (Evans, Handley & Over, 2003; Fugard, Pfeifer, Mayerhofer & Kleiter, 2010; Oberauer & Wilhelm, 2003; Over, Hadjichristidis, Evans, Handley & Sloman, 2007; Pfeifer, 2012; Politzer, Over, & Baratgin, 2010). Ever since the Lewis (1976) triviality arguments, philosophers widely agree that the Equation is incompatible with the classic idea that conditionals are always either true or false

(Adams, 1965, 1998; Edgington, 1995), but see Douven and Verbrugge (2013) for a different view. Indeed, there is plenty of psychological evidence for the so-called ‘defective truth-table’, which is better described as the ‘de Finetti table’ (Baratgin, Over & Politzer, 2013; Evans & Over, 2004; Politzer et al., 2010). People predominantly judge cases where p is known to be false as irrelevant to the truth or falsity of the indicative conditional. For followers of de Finetti (1936), the indicative conditional *If* p , q is ‘void’ in these cases and so is neither true nor false (see Politzer et al., 2010, and Gilio & Over, 2012, on the relation between the indicative conditional and the counterfactual when p is known to be false).¹

Of course, the choice between binary and probabilistic approaches ultimately depends on which theory best accounts for the full range of data, and this includes how people reason with conditionals and how they interpret more expressive conditionals. For a large part, these data still need to be uncovered (Goodwin, in press; Milne, 2012). Prima facie, iterated conditionals, which contain a conditional as one of their constituents, ought to be of particular interest, since they are one of the main drivers of the controversies about the meaning of conditionals. An example is (1), which embeds a conditional at its right:

- (1) If John takes a taxi, then if there is no traffic jam, he will be on time.

The ‘defective’ truth-table has been extended to deal with iterations (de Finetti, 1936, was the first to do this, and see Baratgin et al., 2013), but it is an open question whether we get this response pattern in a truth table task. Even if we do, this does not settle the probability of an iterated conditional, $P(\textit{If } p, \textit{ then if } q, r)$. For assuming the Equation, this probability should amount to $P(\textit{if } q, r|p)$, but this is undefined in standard probability theory when $P(\textit{if } q, r) =$

¹ Although widely used in psychology, the term ‘defective’ is a misnomer. It implies that people who fail to conform to a binary classification in truth table tasks are somehow mistaken. But this need not be the case. In fact, philosophers have proposed that two truth-values are probably not enough for indicative conditionals as early as 1935, the date of de Finetti’s (1936) communication in Paris. See Wason (1966) and Johnson-Laird and Tagart (1969) for the first empirical findings that people classify false antecedent cases as irrelevant.

$P(q/p)$. Some philosophers have argued that the Equation can be extended, so that $P(\text{If } p, \text{ then if } q, r) = P(r|p \text{ and } q)$ (Gibbard, 1981; Edgington, 1995). As an alternative, others have argued that the binary material conditional interpretation familiar from classical logic reappears in embedded conditionals, so that $P(\text{If } p, \text{ then if } q, r) = P(\text{If } p, \text{ then either not } q \text{ or } r)$ (Lowe, 1987). If this turns out to be right, it would shed new light on the choice between the new, probabilistic paradigm and the traditional binary one.

Summing up, iterated conditionals are an important topic in philosophical logic. Yet they are underrepresented in empirical studies.² Two notable exceptions are Douven and Verbrugge (2013) and Huitink (2012). Douven and Verbrugge asked people to suppose that p and then judge the probability of a real world conditional *If* q , r (which might be how they judge the probability of an iterated conditional *If* p , *then if* q , r), and found that their judgments were lower than when they were asked to suppose p *and* q and then estimated the probability of r . Huitink shows that Modus Ponens, i.e. the inference from *If* p , *then if* q , r and p to *If* q , r can be strong or weak, depending on how p is contextually justified. The present paper aims to decrease the dearth of empirical data on iterated conditionals, by directly investigating them in exactly those tasks that inspired the new paradigm: the veteran truth table task and the probability task.

2. Theories of conditionals

Theories of conditionals can roughly be divided into three families. According to the Material conditional interpretation (MC), conditionals are truth-functional and *If* p , q is true when p is false or q is true (Frege, 1879). The mental model theory of Johnson-Laird and Byrne (2002) arguably belongs to this family: conditionals are evaluated by constructing extensional mental models, which correspond to the material conditional, *if all possibilities*

² It is telling that the collection by Oaksford & Chater (2010), which brings together developments in the psychology of reasoning over the last decade or two, contains no contribution about reasoning with iterated hypotheses.

are fleshed out.

In contrast, the Stalnaker-Lewis conditional (SLC) states that conditionals are not truth-functional, but they make crucial reference to alternative states of affairs or possible worlds. They have the following truth conditions: *If p, q* is true if *q* is true in the closest *p*-world(s) (the world(s) that make(s) *p* true that differs minimally from the actual world (Lewis, 1973; Stalnaker, 1968)).³

Finally, the Adams conditional (AdC) does not express a (full) proposition. In its more moderate version, this theory can be traced back to de Finetti (1936) and states that *If p, q* is only true or false in case *p* is true, and if so, its semantic value equals that of *q* (Adams, 1965, 1998; Bennett, 2003; de Finetti, 1936; Edgington, 1995; Politzer et al., 2010). The predictions of these three theories, for each distribution of the semantic values between *p* and *q* are summarized in table 1. Several psychological theories take Adam's seminal work as their starting point. Prominent examples include the Bayesian account by Oaksford and Chater (2009) and theories that take conditionals to express a 'conditional event' (e.g. Fugard et al., 2011; Pfeifer & Kleiter, 2009).

-----Insert table 1 about here-----

In recent years, psychologists have gathered strong empirical evidence against the material conditional. Two prominent paradigms are truth-table tasks and probability tasks. In truth-table tasks, participants are asked to evaluate *If p, q* for different distributions of semantic values of *p* and *q*. As said above, the main result in these experiments is that people

³ There are some noteworthy differences between the proposals by Stalnaker and Lewis. First, Lewis intended this semantics only for counterfactual conditionals *If p had been the case, q would have been the case*. Second, while Stalnaker (1968) originally assumed that there always is a unique closest world, Lewis permits that there are ties. Stalnaker (1980) later admitted that his uniqueness assumption was not warranted, and recast his theory in a supervaluations approach.

do not give the material conditional truth table, but rather the ‘defective’, de Finetti truth table that allows gaps when p is false (see most recently Baratgin et al., 2013, and Politzer et al., 2010).

In probability tasks, participants rate the probability of *If p , q* in a range of frequency distributions. Most people will give the conditional probability of q given p (Douven & Verbrugge, 2010; Evans et al., 2003; Fugard et al., 2011; Oberauer & Wilhem, 2003; Over et al., 2007; Politzer et al., 2010). Some give the probability of p and q , which some view as a simplifying strategy of people of lower cognitive ability (Evans & Over, 2004). The probability of the material conditional hardly shows up in the responses. The Adams conditional seems to offer the most direct explanation of the results in these particular two tasks, certainly in comparison to mental model theory. The latter theory does allow interpretations to diverge from the material conditional (see Johnson-Laird & Byrne, 2002 for details), but only indirectly, by invoking additional principles that have to do a lot of work and that appear to lack independent justification. However, Evans and Over (2004) argue that it would be too rash to conclude that there is a clear win for the Adams conditional, compared to the Stalnaker-Lewis conditional. This is because the probabilities implied by the SLC and the AdC can be too close to detect a difference in experiments that are easy to design (though they are not close in all logically possible cases).

3. Paradigm and predictions

In our experiment, we combined a conditional probability task with the veteran truth-table task, based on the paradigm by Politzer et al. (2010). Participants were presented with abstract iterated conditionals describing chips, and a betting vignette in which two speakers place a bet on a conditional. They were then asked to evaluate the probability of winning the bet, and to give the outcome of the bet for several truth-table conditions, see Fig. 1 for a

sample item.

-----Insert figure 1 about here-----

3.1 Predictions for the truth-table task

There are 8 truth table cases for iterated conditionals with p , q and r as components. However, not all of these are actually necessary to establish a clear response patterns. In fact, it is possible to use only five truth-table cases, which we chose carefully to maximize differential predictions following each theory of conditionals. These predictions are summarized in table 2. Besides the theories MC, SLC and AdC, we also give the predictions in case *If p , then if q , r* is taken as the conjunction between p , q , and r , because we know from experiments on simple conditionals that people will sometimes respond with the probability of the conjunction when abstract materials are used, and we give the predictions for Lowe's (1987) hybrid conditional.

-----Insert table 2 about here-----

Let us now go over these predictions in detail. According to the Material conditional, the semantic value of *If p , then if q , r* is exclusively determined by the semantic values of p , q , and r . It is only false when p and q are true, but r is false. That is, the sentence is only false if the chip that is drawn turns out to be the large black square. In all other cases, the sentence is true.

Given the Adams conditional, and more specifically, given its three-valued incarnation put forward by de Finetti (1936), *If p , then if q , r* only has a truth-value in case p and q are both true. And if it has a truth-value, this value equals that of r . Thus, the sentence is true

when the drawn chip is the large white square and false when it is the large black square. In the other cases, however, the sentence is neither true nor false.

Mental model theory predicts a wide variation of tables, including MC and AdC. Johnson-Laird and Byrne (2002) proposed that people tend, initially, to construct an explicit model for *If p, q* in which *p & q* holds, while making a ‘mental footnote’ that there are possibilities in which *not-p* holds.⁴ The lack of an explicit model for the latter cases leads (somehow) people to judge the conditional as neither true nor false when presented with a *not-p* case. It is not clear exactly how this story plays out for iterated conditionals *If p, then if q, r*, which contain two antecedents, the outer antecedent *p* and the embedded antecedent *q*. It could be that both antecedents are true in explicit models, or that only the truth of the outer antecedent matters. In the first case, the predictions would match AdC, in the latter case, the predictions would match Lowe’s (1987) hybrid conditional. Additionally, some participants should respond with the MC pattern. Although iterated conditionals are highly complex structures, in our experiment it cannot be too difficult to flesh out the core MC meaning that mental model theory predicts, because there is a constant reminder as to which possibilities to include: the visual display of the collection of chips arguably is a fully explicit model.

We will now turn to the Stalnaker/Lewis closeness conditional. In case either *p* or *q* is false, this theory claims that the truth-value of *If p, then if q, r* depends on the truth-values of *p, q* and *r* in the closest worlds where *p* and *q* are true. For our materials, there is a relatively simple way to determine closeness: by counting the features that the different chips (worlds) have in common (this approach to similarity is also advocated in Tversky, 1977). Chips with two features in common with the actual chip are more similar to it than chips that have one feature in common. For example, the small white square fails to satisfy the antecedent of the

⁴ This is the proposal for ‘basic’ conditionals where basic means ‘with a neutral content that is as independent as possible from context and background knowledge’ (Johnson-Laird & Byrne, 2002, p. 648). Since we only use abstract materials in our experiment, we assume that our target sentences are indeed basic and that the processes of semantic and pragmatic modulation, which are restricted to non-basic conditionals, play no role.

embedded *If it is large, then it is white*. To evaluate the iterated conditional, one thus needs to examine the closest large chip. Since the small white square shares two features (color and shape) with the large white square and only one with the other chips in the distribution, this is the closest chip. This chip satisfies the consequent *it is white*, and therefore the iteration is true.

3.2 Predictions for the probability task

In the probability task, we varied the distribution of the five kinds of chips. The total number of chips always equaled 10. Of the five truth table cases, one had the frequency of 6, and the others of 1. Thus, in the sample item above, the FFF case (i.e. the small black circle) has a frequency of 6. Now, the material conditional and the Stalnaker-Lewis conditional agree that people will provide as the probability the number of chips where Mary wins her bet divided by the total number of chips. Thus, if the distribution is as in the sample item above, the chance that Mary will win is 9 out of 10 according to MC. After all, there is only one chip that falsifies the sentence. In contrast, SLC states that this chance is only 3 out of 10, because the sentence is true only in 3 cases: the large white square, the small white square and the large white circle.

The Adams conditional proposes that the probability of simple conditionals *If p, q* equals the conditional probability of *q* given *p*. However, this cannot be extended to iterated conditionals *If p, then if q, r*. In standard probability theory there is no such thing as the conditional probability of *q* given *r*, given *p*. This is simply undefined. Two solutions have been proposed. First, in a three-valued system one might extend standard probability theory such that a sentence's probability equals the number of cases where it is true, divided by the total number of cases *in which it has a truth-value*, see Rothschild (in press) for details. The predicted answer is then 1 out of 2. Second, some hold that right-nested conditionals are

rephrased as simple conditionals with conjunctive antecedents *If p and q, r* during interpretation (Edgington, 1995; Gibbard, 1981). This rephrasing accords with the logical Import-Export principle:

(2) Import-Export

If p, then if q, r is equivalent to *If p and q, r*

It follows that the chance that Mary wins her bet equals the chance that *If the chip is square and large, it is white* is true. Since only two chips are square and large, of which only one is white, the appropriate answer in our sample item is again 1 out of 2. Import-Export seems to respect the meaning of iterated conditionals, but people might of course employ alternative heuristics to reduce the complexity of the task. The predictions for the probability task are summarized in table 3:

-----Insert table 3 about here-----

Note that the denominator that participants use in their response is highly informative, especially in the first two distributions in table 3. If, for instance, someone responds that the probability of *If p, then if q, r* is 6 out of 7, then this person cannot be interpreting the sentence as a Stalnaker/Lewis conditional. For there is no x between 0 and 10 such that $x/10 = 6/7$.

4. Method

4.1 Participants

A total of 182 volunteers were recruited via the web portal *Psychological Research on*

the Net (<http://psych.hanover.edu/research/exponnet.html>). To preclude random clicking, we used the control page technique recommended by Oppenheimer, Meyvis, and Davidenko (2009): A page that looked like one of the experimental pages, except for two things: (1) At the top of the page the following instructions were given: ‘Please ignore all questions on this screen including the scale below. Instead, proceed to the end of the screen and tick the small box. This is done just to screen out random clicking.’ (2) At the bottom of the page participants were presented with a tick box and the instructions ‘To continue tick the box and then press NEXT.’ Results from participants who failed to tick the box ($n=100$) were excluded from the main analysis.⁵ Out of the remaining 82, we further excluded from analysis results from participants who failed to answer at least half of the experimental questions ($n=8$), participants with prior training in logic ($n=10$), and participants whose main language was not English ($n=6$). Although the attrition rate is high, this is not unusual in unpaid web studies, and, importantly, the remaining sample ($n=58$, 36 male) did not differ significantly from excluded participants in sex, age, and education level. Mean age was 27. 35 participants had secondary school education or equivalents, and 17 had an academic degree.

4.2 Design and materials

We used a 5 x 2, entirely within design, with distribution (61111, 16111, 11611, 11161, 11116) and conditional type (iterated, imported) as independent variables. We used the 5

⁵ As suggested by Andy Fugard (p.c.), it would be interesting to see what kind of answers the people who failed to respond to the control question gave. However, only 10 of these participants also responded to at least one test page. Of these, three responded to a single test page, one responded to two test pages, five responded to three test pages, and one responded to four test pages (out of a total of ten test pages). Looking at the response pattern overall, we have a total of 26 response patterns to the truth table task across the ten participants. Out of these, 13 patterns (50%) were conjunctive, 7 (27%) were AdC, and 6 (23%) were unclassified. Similarly, we have a total of 25 response patterns to the probability task across the ten participants, and out of these, 10 (40%) were conjunctive, 3 (12%) were AdC, 11 (44%) were unclassified, and a single one (4%) was MC. The latter is most probably a fluke, especially since all other response patterns produced by this participant were unclassified. The prevalence of conjunctive and unclassified responses is in line with the standard explanation of these responses as representing a low level of cognitive effort.

truth table cases with 5 frequency distributions described in Table 3, and compared the iterated form *If p, then if q, r* with the imported form *If p and q, r*. Each web page presented both tasks (truth table task and probability task), with the probability task always presented first, followed by the five truth table tasks (TTT, TTF, TFT, FTT, and FFF) in random order.⁶ Pages were presented in individually randomized order. The dependent variables were the numerator and the denominator responses for the probability task, and, for the truth table task, the categorical responses win (T) / lose (F) / bet called off (#).

4.3 Procedure

Participants were presented with an information and consent page followed by a demographics page. They were then given two practice pages containing similar materials to those in the experimental pages. Ten experimental pages followed, with the control page always inserted between the fifth and the sixth experimental pages.

5. Results and discussion

5.1 Truth table task

There were hardly any differences across distributions (nor did we predict any, for the truth table task), so we report aggregated percentages. We computed truth table response patterns by combining responses across all five truth table cases, TTT, TTF, TFT, FTT, and FFF respectively. For example, participants who evaluated TTT as T, TTF as F, and all other truth table cases as #, were classified as displaying a TF### pattern – i.e., an Adams conditional pattern.

⁶ We kept the order of the tasks constant as carryover effects from the probability task to the truth-table task were much more likely than the other way around. Asking about the chance that Mary wins her bet, after the chip has been drawn and revealed to be a small white circle would turn the task into judging the probability of the corresponding counterfactual ‘Had the chip been square, then if ...’.

-----Insert table 4 about here-----

As can be seen in Table 4, slightly more than half of the responses display a clear TF### pattern – that is, the AdC response. Although there are somewhat more AdC responses for the iterated form (*If p, then if q, r*) relative to the imported form (*If p and q, r*), the difference was not significant. A series of McNemar-Bowker tests done separately on the response pattern for each distribution, comparing the iterated to the imported form, found no significant differences ($\chi^2(1) \leq 8.8$, $p > .05$). About a fifth of the responses display the TFFFF pattern consistent with a conjunctive response.

To check within-participant consistency, we counted the number of AdC responses across the five iterated items and across the five imported items respectively. This generated two 0-5 scores, measuring consistency in AdC responses within the iterated form and the imported form respectively. A high score reflects high AdC consistency. We also generated two such consistency scores for conjunctive responses. A paired-samples t test revealed that the number of AdC responses for the imported forms was significantly higher than for the iterated forms ($M=3.0$ vs $M=2.7$ respectively), ($t(57)=2.296$, $p=.025$, Cohen's $d=.61$, $r=.29$), indicating a difference in consistency of AdC responding between these test conditions. We attribute this difference to the relative difficulty of the iterated form generating more noise. No difference in consistency was found between the conjunctive consistency scores.

Unclassified responses could, in theory, be SLC, MC or Lowe responses, but the data suggest otherwise. The TTT and TTF cases are trivial – all theories of conditionals evaluate them as T and F respectively. Thus, any meaningful patterns should conform at least to the general outline of TF___ where ‘_’ can take any value. Looking at all unclassified response patterns, we only found 14 response patterns that conformed to this rule: All other patterns

violated it. To put things in proportion, we have a total of 580 response patterns across all participants and all distributions, out of which 113 were unclassified. Thus, even if these 14 patterns somehow reflect one of the further predicted patterns (which we doubt), they constitute a negligible minority. Many of the unclassified patterns were simply the result of missing values. Hence, we interpret the unclassified response patterns as a result of typing errors, missing values, and (despite our best efforts to prevent it) some random clicking.

A small minority of participants displayed a curious TFF## pattern, which we had not anticipated. Although the pattern was not very frequent, it was too consistent to be dismissed as just noise. One possibility is that it reflects an alternative parsing of the target sentences as *If p then (q and r)*, where the embedded conditional is replaced by a conjunction. This alternative parsing is not equivalent to the iterated form *If p then if q r*, but the use of abstract materials may have obscured the difference in meaning. Although such parsing would be easier to process relative to the iterated form, there is no processing gain relative to the imported form (although no processing penalty either).

One last finding worth mentioning is that participants with the AdC pattern were fairly consistent. 53% of the participants displayed an AdC pattern across at least 8 out of 10 patterns, and 36% consistently did so for all ten.

5.2 Probability task

For each participant we computed ten conditional probability scores by dividing the numerator by the denominator for each of the probability questions. With participants with missing values in either numerator or denominator excluded, we had a sample of 40 participants for this analysis. We analyzed the results using a 5x2 ANOVA with distribution (61111, 16111, 11611, 11161, 11116) and conditional type (iterated, imported) as within-participants variables. The results showed a main effect of distribution ($F(2,85)=275.7$,

MSE=.027, $p<.001$, $\eta_p^2=.88$), but not of conditional type ($F(1,39)=2.73$, MSE=.009, $p>.05$) or interaction between conditional type and distribution ($F(2,81)=.71$, MSE=.010, $p>.05$).⁷

To take individual differences into account, we used the pattern data taken from the truth table task. Recall that 36% of participants displayed a consistent Adams conditional pattern across all evaluations. We therefore split our sample into AdC responders ($n=21$) and non-AdC responders ($n=19$), and re-ran our analysis with AdC response pattern as the between-participants variable, using a 6x2x2 mixed ANOVA, with 6 levels of distribution and 2 levels of conditional type as within-participants variables, and 2 levels of individual differences (AdC) as a between-participants variable.

In addition to the main effect of distribution ($F(3,93)=313$, MSE=.021, $p<.001$, $\eta_p^2=.89$), there was also a main effect of individual differences ($F(1,38)=466$, MSE=.133, $\eta_p^2=.93$), which was explained by an interaction between distribution and individual differences ($F(3,93)=6.6$, MSE=.021, $\eta_p^2=.15$). The means, displayed in Table 5, closely resemble the predictions for AdC and conjunctive probability responses respectively.

-----Insert table 5 about here-----

As is clear from Table 5, probability estimates of participants who displayed a consistent AdC pattern in the truth table task closely resemble the predictions for AdC in the probability task. That is, the response pattern generalized across tasks. Probability estimates of the rest of the participants are a good match for the conjunctive predictions, although with slightly larger discrepancy, probably due to noise and the possible inclusion of some TFF## responders in this group. Although there are not many data points to compare, it is worth noting that the overall correlation between the observed means and the relevant probability

⁷ Where sphericity was violated we report degrees of freedom corrected by the Greenhouse-Geisser adjustment.

predictions was $r=.935$ ($n=10$, $p<.01$).

5.3 Discussion

Our findings suggest that iterated conditionals *If p , then if q , r* are evaluated as neither true nor false in case the outer antecedent p or the inner antecedent q is false. As for the probability of iterated conditionals, our results indicate that people judge the probability of *If p , then if q , r* as the conditional probability of r , given p and q . The two most important findings for simple conditionals, i.e. the defective truth table and the conditional probability hypothesis, thus extend to iterated conditionals. We have also found that some people will treat the conditional as the conjunction between p , q and r , both in the truth-table task and in the probability task. This again replicates individual differences typically found in these tasks with abstract simple conditionals (Evans et al., 2003; Wilhelm & Oberauer, 2003).

The present study goes beyond the conclusions one can draw from experiments with simple conditionals. As mentioned above, when restricted to simple conditionals, the probability task cannot distinguish between SLC and AdC, because the probabilities predicted by these theories can be very close in experiments that are easy to design. In the case of iterated conditionals, however, the predictions diverge considerably. Therefore, the results in our probability task univocally support AdC. Nothing like SLC was found, nor did we find any MC responses. Though the psychology of conditionals may well be so complex that no theory is able to account for the full range of data, these particular results do speak against mental model theories of conditionals. We went over the data with a fine-toothed comb, we even went to the length of examining individual ‘other’ patterns, and still found no evidence for the material conditional. Finally, it is worth mentioning that there is no evidence for a hybrid interpretation of conditionals either. If, as Lowe (1987) claims, conditionals turn into material conditionals when they are embedded in the consequent of a

conditional, we should have found that '*If p, then if q, r*' is true in case *p* is true but *q* is false. But our participants agree that it is neither true nor false in this case. Embedding a conditional thus does not appear to influence its interpretation. Instead, our results suggest that iterated conditionals are not all that different from their simple cousins.

We did not find any significant difference between the iterated form *If p, then if q, r* and the imported form *If p and q, then r*. This seems to contradict the recent study by Douven and Verbrugge (2013), in which people associated a lower probability with the iteration *If p then if q, r* than with the imported form *If p and q, r*. However, their study differed from ours in that thematic materials were used and participants had to rely on their world knowledge to estimate probabilities rather than on given frequencies. It therefore seems likely that the participants in Douven and Verbrugge's study were more influenced by pragmatic considerations than those in our experiment, and it is thus open to debate whether *If p then if q, r* is semantically equivalent to *If p and q, r*, while the two forms have different use conditions. To be sure, drawing the boundaries between semantics and pragmatics is a non-trivial task, and theories within the AdC family need not make the same choices.⁸ An alternative explanation of Douven and Verbrugge's results is that people are generally more cautious when assigning probabilities to conditionals versus categorical sentences.

Finally, although our findings suggest that the iterated form and the imported form are equivalent, our results say nothing about Gibbard's (1981) thesis that *If p, then if q, r* is rephrased as *If p and q, r* during interpretation. It is still unclear whether this happens at all, and if it does, whether the iteration just cannot be interpreted before it is rephrased, as Gibbard claims, or whether the rephrasing happens because replacing conditionals with

⁸ Another result that seems to go against the Equation is reported by Fugard, Pfeifer, & Mayerhofer (2011). Though individuals assign *If x is 2 then x is even* the conditional probability, when the consequent is expressed as a disjunction rather than a hypernym, *If x is 2, then x is 2 or x is 4*, they tend to give probability 0. The authors suggest adding a relevance criterion to the notion of logical consequence. Arguably this 'semanticizes' a pragmatic concept, blurring the semantics/pragmatics boundary.

conjunctions is a general strategy employed when individuals are driven to reduce processing load. We suspect that this latter view is correct and that it also underlies the minority TFF## - pattern that we found in the truth table task. At any rate, this curious minority pattern provides another example for theories of conditionals to go off and explain. Note that it is unclear how this interpretation could be explained in terms of (forgetting of) the mental footnotes postulated by mental model theory.

6. Conclusion

The present study replicated and extended the individual differences observed in simple conditionals to more expressive conditionals. The results strongly support the Adams conditional, and the new paradigm that takes this conditional as a starting point. Interestingly, participants were shown to reason similarly for iterated conditionals *If the chip is square, then if it is large, it is white* as for the imported variant *If the chip is square and large, it is white*. Finally, a new interpretation was uncovered, which we suggested is due to interpreting *If p, then if q, r* as *If p, then q and r*. Although iterated conditionals are fiendishly difficult, it is well worth exploring them in empirical studies, for iterated conditionals allow us to decide between SLC and AdC, which is not possible with simple conditionals.

References

- Adams, E. W. (1965.) A logic of conditionals. *Inquiry* 8, 166-97.
- Adams, E. W. (1998). *A primer of probability logic*. Stanford: CLSI publications.
- Baratgin, J., Over, D. E., & Politzer, G. (2013). Uncertainty and the de Finetti tables. *Thinking & Reasoning*, 19, 308-328.
- Bennett, J. (2003). *A philosophical guide to conditionals*. Oxford: Clarendon Press.
- de Finetti, B. (1936). La logique de la probabilité, in *Actes du Congrès International de Philosophie Scientifique a Paris 1935*, Tome IV, Hermann et Cie, Paris, 1936, pp. 1- 9. Translated into English by R. B. Angell, The logic of probabilities. *Philosophical Studies* 77, 181-190.

- Douven, I. & Verbrugge, S. (2013). The probabilities of conditionals revisited. *Cognitive Science*, doi: 10.1111/cogs.12025.
- Douven, I. & Verbrugge, S. (2010). The Adams family. *Cognition*, 117, 302-318.
- Edgington, D. (2008.) Conditionals. In E. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Winter 2008 Edition)*. <http://plato.stanford.edu/archives/win2008/entries/conditionals/>
- Edgington, D. (1995). On conditionals. *Mind* 104(414), 235-329.
- Elqayam, S. & Over, D. E. (2013). New paradigm psychology of reasoning: An introduction to the special issue. *Thinking & Reasoning*, 19, 249-265.
- Evans, J.St.B.T. (2012). Questions and challenges for the new psychology of reasoning. *Thinking and Reasoning*, 18, 5-31.
- Evans, J.St.B.T. & Over, D. E. (2004). *If: supposition, pragmatics and dual processes*. Oxford: Oxford University Press.
- Evans, J.St.B.T., Handley, S. and Over, D. E. (2003). Conditionals and conditional probability. *Journal of Experimental Psychology: Learning, Memory and Cognition* 29, 321-355.
- Frege, G. (1879). Begriffsschrift. Reprinted in: Gottlob, F. (1964). *Begriffsschrift und andere Aufsätze*. Hildesheim: Georg Olms Verlagsbuchhandlung.
- Fugard, A., Pfeifer, N., Mayerhofer, B. (2011). Probabilistic theories of reasoning need pragmatics too: Modulating relevance in uncertain conditionals. *Journal of Pragmatics* 43, 2034-2042.
- Fugard, A., Pfeifer, N., Mayerhofer, B. and Kleiter, G. (2011). How people interpret conditionals: Shifts towards the conditional event. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 37(3), 635–648.
- Gibbard, A. (1981). Two recent theories of conditionals. In W.L. Harper, R. Stalnaker & G. Pearce (Eds.), *Ifs: conditionals, belief, decision, change and time* (pp. 211–247). Dordrecht: Reidel.
- Gilio, A., & Over, D.E. (2012). The psychology of inferring conditionals from disjunctions: A probabilistic study. *Journal of Mathematical Psychology*, 56, 118–131.
- Goodwin, G.P. (In press). Is the basic conditional probabilistic? *Journal of Experimental Psychology: General*.
- Johnson-Laird, P.N. and Byrne, R.M.J. 2002. Conditionals: a theory of meaning, pragmatics, and inference. *Psychological Review*. 109(4), 646-678.

- Johnson-Laird, P.N. and Tagart, J. 1969. How implication is understood. *The American Journal of Psychology*. 82(3), 367-373.
- Huitink, J. (2012). McGee's counterexample to modus ponens in context. In B. Stolterfoht and S. Featherston (Eds.), *Empirical approaches to linguistic theory: Studies of meaning and structure* (pp. 169-185). De Gruyter Mouton.
- Lewis, D. K. (1973). *Counterfactuals*. Cambridge, USA: Harvard University Press.
- Lowe, E.J. (1987). Not a counterexample to modus ponens. *Analysis* 47(1), 44-47.
- Milne, P. (2012). Indicative conditionals, conditional probabilities, and the 'defective truth-table': A request for more experiments. *Thinking & Reasoning*, 18(2), 196-224.
- Oaksford, M. & Chater, N. (2010). *Cognition and conditionals: Probability and logic in human thinking*. New York: Oxford University Press.
- Oaksford, M. & Chater, N. (2009). Précis of Bayesian rationality: The probabilistic approach to human reasoning. *Behavioral and Brain Sciences* 32, 69-120.
- Oberauer, K. & Wilhelm, O. (2003). The meaning of conditionals: conditional probabilities, mental models and personal utilities. *Journal of Experimental Psychology: Learning, Memory, and Cognition* 29(4), 680-693.
- Oppenheimer, D. M., Meyvis, T., & Davidenko, N. (2009). Instructional manipulation checks: Detecting satisficing to increase statistical power. *Journal of Experimental Social Psychology*, 45(4), 867-872.
- Over, D. E. (2009). New paradigm psychology of reasoning. *Thinking & Reasoning*, 15, 431-438.
- Over, D. E. & Hadjichristidis, C. & Evans, J.St.B.T. & Handley, S. & Sloman, S. (2007). The probability of causal conditionals. *Cognitive Psychology* 54, 62-97.
- Pfeifer, N. (2012). Experiments on Aristotle's Thesis: Towards an experimental philosophy of conditionals. *The Monist* 95(2), 223-240.
- Pfeifer, N. & Kleiter, G.D. (2009). Framing human inference by coherence based probability logic. *Journal of Applied Logic* 7(2), 206-217.
- Politzer, G., Over, D. E. & Baratgin, J. (2010). Betting on conditionals. *Thinking & Reasoning* 16(3), 172-197.
- Rothschild, D. (In press). Capturing the relationship between conditionals and conditional probability with a trivalent semantics. *Journal of Applied Non-Classical Logics*.
- Stalnaker, R. (1980). A defence of Conditional Excluded Middle. In W.L. Harper, R. Stalnaker & G. Pearce (Eds.), *Ifs: conditionals, belief, decision, change and time* (pp. 87-104). Dordrecht: D. Reidel

Publishing Company.

Stalnaker, R. (1968). A theory of conditionals. In N. Rescher (Ed.), *Studies in Logical Theory*. Oxford: Blackwell.

Tversky, A. (1977). Features of similarity. *Psychological Review* 84(4), 327-352.

Wason, P. C., Reasoning, in B. Foss (ed.), *New horizons in psychology* (pp. 135-151). Harmondworth: Penguin.

Table 1: Predictions for If p, q for each possible distribution of the truth-values between p and q.

	MC	SLC	AdC
TT	T	T	T
TF	F	F	F
FT	T	T/F	#
FF	T	T/F	#

Fig. 1 Sample item.

This drawing represents a collection of chips:



A random chip is to be chosen in a fair way. Mary tells Peter:

“I bet you £1 that if the chip is square then if it is large, it is white”.

Each of them puts £1 on the table. They agree that the winner will pocket the entire stake.

(Probability task:)

What are the chances that Mary wins her bet? ____ out of ____

(Truth-table task:)

Suppose the chip is square, small and white. Do you think that:

Mary wins her bet/Mary loses her bet/The bet is called off

Table 2, Truth table task predictions for If the chip is square, then if it is large, it is white.






		MC	SLC	AdC	Conj.	Lowe
	TTT	T	T	T	T	T
	TTF	F	F	F	F	F
	TFT	T	T	#	F	T
	FTT	T	T	#	F	#
	FFF	T	F	#	F	#

Table 3, Probability task predictions for If the chip is square, then if it is large, it is white.

Distribution of TTT, TTF, TFT, FTT, and FFF cases	MC	SLC	AdC	Conj.	Lowe
61111	9/10	8/10	6/7	6/10	6/8
16111	4/10	3/10	1/7	1/10	2/8
11611	9/10	8/10	1/2	1/10	6/8
11161	9/10	8/10	1/2	1/10	2/3
11116	9/10	3/10	1/2	1/10	2/3

Table 4: Truth table pattern percentages by conditional type, averaged across distributions

Pattern	<i>If p, then if q, r</i>	<i>If p and q, r</i>
TF### (AdC)	54%	60%
TFFFF (conjunctive)	20%	19%
TFF## (<i>If p, then q and r</i>)	5%	6%
Unclassified	21%	18%

Table 5: Mean probability estimates as function of distribution and individual differences pattern (n=40)

	Distribution	Mean	(SE)	AdC Prediction	Conjunctive prediction
Non-AdC Ps (n=19)	61111	.69	(.029)		.60
	16111	.12	(.007)		.10
	11611	.28	(.035)		.10
	11161	.27	(.037)		.10
	11116	.29	(.037)		.10
AdC Ps (n=21)	61111	.82	(.027)	.86	
	16111	.14	(.007)	.14	
	11611	.46	(.034)	.50	
	11161	.44	(.035)	.50	
	11116	.44	(.035)	.50	